THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH3070 Introduction to Topology 2017-2018
Tutorial Classwork 3

1. Let $(X, \mathfrak{T}_X)$ and $(Y, \mathfrak{T}_Y)$ be two topological space and $f : (X, \mathfrak{T}_X) \to (Y, \mathfrak{T}_Y)$.

   We say that f is *sequentially continuous* if for any sequence $x_n \to x$, we have $f(x_n) \to f(x)$.

   (a) Consider the cocountable topology of $\mathbb{R}$. Show that if $x_n \to x$, then there exists $N \in \mathbb{N}$ such that $x_n = x$ for all $n \geq N$.

   (b) By a), find a function f that is sequentially continuous but not continuous.

   (c) * Show that if X is C_I and f is sequentially continuous, then f is continuous.

2. Let $(X, \mathfrak{T})$ be a topological space and $C \subset X$ be a closed subset of X. Show that C is nowhere dense if and only if $C = \overline{U} \cap \overline{X \backslash U}$ for some open set U.

   (That means a closed nowhere dense set is the frontier (or boundary) of some open set.)

3. Show that a topological space X is of second category if and only if any countable intersection of open dense subset of X is non-empty.

   (Hint: $A \subset X$ is open dense if and only if $X \backslash A$ is closed nowhere dense.)